Reporter 1. According to the sign convention discussion in subsection **2.1** of my paper vector bundle $\mathcal{E}$ is ample/nef iff $\mathcal{O}_W(1)$ is ample/nef on $W := \mathbb{P}(\mathcal{E}^\vee)$ for the dual $\mathcal{E}^\vee$ (one can see this from the Hirsch formula by restricting to generic curve on S). This is a standard notational abuse (which I tried to stress on in Remark 2.2). Let me repeat that there is no canonical choice for π a priori, hence for $\mathcal{O}_W(1), \pi_*\mathcal{O}_W(1)$, etc. – these are defined a posteriori by K_W, Hirsch-type relations and other intrinsic properties of W (it is unfortunate tradition to assume that there is a natural choice for π – aka of local trivialization for $\mathcal{E}$).[1)]

By the same sign convention any $\mathcal{E} \otimes \mathcal{O}_S(D)$ is ample/nef iff $\mathcal{O}_W(1)$ is this on $W := \mathbb{P}(\mathcal{E}^\vee \otimes \mathcal{O}_S(D))$ (any $D \in \operatorname{Pic} S$). The latter is because $\mathcal{E}, \pi$ are fixed, and the group $\pi^* A^1(S)$ does not change under any (non-canonical) isomorphisms $\mathbb{P}(\mathcal{E}) \simeq \mathbb{P}(\mathcal{E}^\vee) \simeq \mathbb{P}(\mathcal{E}^\vee \otimes \mathcal{O}_S(D))$ over S.

When applied to $\mathcal{E} \otimes \mathcal{O}_S(\frac{n-1}{2} K_S)$ we find that $-K_W$ being nef is not controversial: for instance $\mathcal{E}^\vee \otimes \mathcal{O}_S(\frac{n-1}{2} K_S)$ has positive degree (also one can not simply dualize the sequence (2.3) in my paper to obtain a surjection of $\mathcal{E}^\vee \otimes \mathcal{O}_S(\frac{n-1}{2} K_S)$ onto a negative sheaf).

[1)] As for Chern classes, most naturally they are defined (by Chern) for *complex* vector bundles in terms of curvature tensor, its Weil polynomials, and so on.